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Bounds on Neutral Current Interactions In Weak Pion Production

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ABSTRACT

Bounds on the cross sections for both exclusive and inclusive weak pion production via neutral currents are obtained for the Weinberg model of weak and electromagnetic interactions. The bounds derived involve no assumptions at the outset about the relative importance of the I = 1/2 and I = 3/2 amplitudes. In lieu of sufficient experimental information, numerical estimates are given which incorporate $\Delta(1236)$ dominance and yield $\sigma(\nu p \to \nu \pi^+ n)/\sigma(\nu p \to \mu^- \pi^+ p) \gtrsim 0.03$ and $\sigma(\nu \bar{N} \to \nu \pi^0 \bar{N})/\sigma(\nu n \to \mu^- \pi^0 p) \gtrsim 0.19$.

Several papers have appeared in the literature recently which attempt to place lower bounds on cross sections that would involve 1st order weakly-coupled neutral currents in the Weinberg model[1] of weak and electromagnetic interactions. The cross section ratios investigated include the elastic lepton interactions[2] $\nu_e + e \rightarrow \nu_e + e$ and $\nu_\mu + e \rightarrow \nu_\mu + e$, the elastic semileptonic interactions[3] $\nu_\mu + N \rightarrow \nu_\mu + N$, inelastic weak pion production[3,4] $\nu_\mu + N \rightarrow \nu_\mu + \pi + N$, and the inclusive reaction[5,6] $\nu_\mu + N \rightarrow \nu_\mu$ + anything.

The weak pion production bounds are of immediate interest to the experimentalist, but the bounds obtained invoke the assumptions of $\Delta(1236)$ dominance.[7] The purpose of this note is to relax this assumption and to deduce bounds for both the exclusive weak pion production process

$$\nu_\mu + N \rightarrow \nu_\mu + \pi + N \tag{1}$$

and for the inclusive weak pion production process

$$\nu_\mu + N \rightarrow \nu_\mu + \pi + \text{anything}. \tag{2}$$

The neutral current of interest will be written in the form

$$J_\lambda^{(0)} = A_\lambda^3 + x V_\lambda^3 + y V_\lambda^0 , \tag{3}$$

where $y = -2 \sin^2 \theta_W$ and $x = 1 + y$ in the Weinberg model. The Weinberg angle θ_W has been bounded by $\sin^2 \theta_W \lesssim 0.33$ in Ref. 2 by allowing one standard deviation in the experimental data. To be conservative in what follows, we shall take

$$\sin^2\theta_W \lesssim 0.40, \tag{4}$$

which corresponds to[2]

$$\frac{\sigma(\bar{\nu}_e + e \to \bar{\nu}_e + e)_{expt}}{\sigma(\bar{\nu}_e + e \to \bar{\nu}_e + e)_{V-A}} \lesssim 3.$$

A. EXCLUSIVE PROCESS

1. Bounds Involving Conventional Processes

Any weak pion process of type (1), or its charged current counterpart, can be expressed in terms of one (I = 1/2) V^0 amplitude, two (I = 1/2 and I = 3/2) V^3 amplitudes, and two (I = 1/2 and I = 3/2) A^3 amplitudes. If one averages over the proton and neutron targets in the neutral current interactions to eliminate the isovector-isoscalar interference term, one finds two sets of relations for the structure functions. We list the results here for the vector, interference, and axial vector contributions:

$$\begin{aligned} &W^V(\nu p \to \nu \pi^+ n) + W^V(\nu n \to \nu \pi^- p) \\ &\quad = y^2\left[W(ep \to e\pi^+ n) + W(en \to e\pi^- p)\right] + (x^2 - y^2) W^V(\nu n \to \mu^- \pi^0 p), \end{aligned} \tag{5a}$$

$$W^I(\nu p \to \nu \pi^+ n) + W^I(\nu n \to \nu \pi^- p) = x\, W^I(\nu n \to \mu^- \pi^0 p), \tag{5b}$$

$$W^A(\nu p \to \nu \pi^+ n) + W^A(\nu n \to \nu \pi^- p) = W^A(\nu n \to \mu^- \pi^0 p), \tag{5c}$$

$$W(ep \to e\pi^+ n) + W(en \to e\pi^- p) = \frac{16}{9}|E_s|^2 + W^V(\nu n \to \mu^- \pi^0 p), \tag{5d}$$

$$\begin{aligned} & W^V(\nu p \to \nu\pi^0 p) + W^V(\nu n \to \nu\pi^0 n) \\ & \quad = y^2\left[W(ep \to e\pi^0 p) + W(en \to e\pi^0 n)\right] \\ & \qquad + \tfrac{1}{2}(x^2 - y^2)\left[W^V(\nu p \to \mu^-\pi^+ p) + W^V(\nu n \to \mu^-\pi^+ n) - W^V(\nu n \to \mu^-\pi^0 p)\right], \end{aligned} \tag{6a}$$

$$\begin{aligned} & W^I(\nu p \to \nu\pi^0 p) + W^I(\nu n \to \nu\pi^0 n) \\ & \quad = \tfrac{1}{2}x\left[W^I(\nu p \to \mu^-\pi^+ p) + W^I(\nu n \to \mu^-\pi^+ n) - W^I(\nu n \to \mu^-\pi^0 p)\right], \end{aligned} \tag{6b}$$

$$\begin{aligned} & W^A(\nu p \to \nu\pi^0 p) + W^A(\nu n \to \nu\pi^0 n) \\ & \quad = \tfrac{1}{2}\left[W^A(\nu p \to \mu^-\pi^+ p) + W^A(\nu n \to \mu^-\pi^+ n) - W^A(\nu n \to \mu^-\pi^0 p)\right], \end{aligned} \tag{6c}$$

$$\begin{aligned} & W(ep \to e\pi^0 p) + W(en \to e\pi^0 n) \\ & \quad = \tfrac{8}{9}|E_s|^2 + \tfrac{1}{2}\left[W^V(\nu p \to \mu^-\pi^+ p) + W^V(\nu n \to \mu^-\pi^+ n) - W^V(\nu n \to \mu^-\pi^0 p)\right]. \end{aligned} \tag{6d}$$

In Eqs. (5d) and (6d), E_s represents the isoscalar matrix element.

In a manner which is familiar from the work of Paschos and Wolfenstein,[6] one can use Eqs. (5) and (6) to deduce two lower bounds on the neutral current cross sections. We find

$$R_1 \equiv \frac{\sigma(\nu p \to \nu \pi^+ n) + \sigma(\nu n \to \nu \pi^- p)}{2\sigma(\nu n \to \mu^- \pi^0 p)}$$

$$\geq \frac{1}{2}\left[1 - 2\sin^2\theta_W \left(\frac{V_{em}(ep \to e\pi^+ n) + V_{em}(en \to e\pi^- p)}{2\sigma(\nu n \to \mu^- \pi^0 p)}\right)^{1/2}\right]^2, \tag{7}$$

and

$$R_2 \equiv \frac{2\sigma(\nu p \to \nu \pi^0 p) + 2\sigma(\nu n \to \nu \pi^0 n) + \sigma(\nu p \to \nu \pi^+ n) + \sigma(\nu n \to \nu \pi^- p)}{2\sigma(\nu p \to \mu^- \pi^+ p) + 2\sigma(\nu n \to \mu^- \pi^+ n)}$$

$$\geq \frac{1}{2}\left[1 - 2\sin^2\theta_W \left(\frac{2V_{em}(ep \to e\pi^0 p) + 2V_{em}(en \to e\pi^0 n) + V_{em}(ep \to e\pi^+ n) + V_{em}(en \to e\pi^- p)}{2\sigma(\nu p \to \mu^- \pi^+ p) + 2\sigma(\nu n \to \mu^- \pi^+ n)}\right)^{1/2}\right]^2. \tag{8}$$

Here $$V_{em} \equiv \frac{G^2}{\pi} \frac{1}{4\pi\alpha^2} \int q^4 \frac{d\sigma_{em}}{dq^2}\, dq^2.$$

The inequalities above result from dropping the isoscalar contribution to the electromagnetic processes and through use of a Schwarz inequality placed on the vector and axial vector contributions.

We emphasize that the above bounds involve no assumptions about the relative importance of the I = 1/2 and I = 3/2 amplitudes. In time one will be able to test these bounds experimentally. In the meantime, however, we shall now use $\Delta(1236)$ dominance to estimate the right-hand sides. We use the data of S. Galster, et al.,[8] to estimate

$$V_{em}(ep \to e\Delta^+) = 0.156 \times 10^{-38}\,cm^2,$$

and the new results from the Argonne neutrino experiment which yield[9]

$$\sigma(\nu p \to \mu^- \pi^+ p) \simeq 0.75 \times 10^{-38}\,cm^2,$$

both at E = 2 GeV. These results imply that

$$R_1 \gtrsim 0.15 \tag{9}$$

and

$$R_2 \gtrsim 0.15. \tag{10}$$

Again invoking Δ dominance, we can relate $\sigma(\nu n \to \mu^- \pi^0 p)$ to $\sigma(\nu p \to \mu^- \pi^+ p)$ and with that (9) find that

$$R_3 \equiv \frac{\sigma(\nu p \to \nu \pi^+ n)}{\sigma(\nu p \to \mu^- \pi^+ p)} \gtrsim 0.03. \tag{11}$$

This result should be compared with $R_3 \simeq 0.11$ derived under the assumption of Δ dominance from the outset.[3]

2. Bounds Based on Estimates of the I = 1/2 and I = 3/2 Amplitudes

As an alternative to the procedure used in part 1, we can express the cross section ratios in terms of the I = 1/2 and 3/2 amplitudes and then estimate the relative effect of these amplitudes. This method allows us to deduce additional bounds for other cross section ratios.

In particular, we consider the ratio

$$R_4 \equiv \frac{\sigma(\nu p \to \nu \pi^0 p) + \sigma(\nu n \to \nu \pi^0 n)}{2\sigma(\nu n \to \mu^- \pi^0 p)} \tag{12}$$

for which the Columbia group[10] has placed an upper bound of $R_4 \lesssim 0.14$ (90% confidence level). In terms of the I = 3/2 and 1/2 amplitudes X_3 and X_1, respectively, and the contributions of the isovector and isoscalar electromagnetic current E_v and E_s, we can write

$$R_4 = \frac{|X_3 - \frac{1}{2}X_1 - \gamma E_V|^2 + \gamma^2 |E_S|^2}{|X_3 + X_1|^2}. \tag{13}$$

With no dynamical assumptions whatsoever, we have

$$R_4 \geq \left[\left|\frac{X_3 - \frac{1}{2}X_1}{X_3 + X_1}\right| - \left|\frac{\gamma E_V}{X_3 + X_1}\right| \right]^2 + \gamma^2 \left|\frac{E_S}{X_3 + X_1}\right|^2. \tag{14}$$

The second term in the square bracket can be evaluated according to Ref. 6 and is bounded by

$$\left|\frac{\gamma E_V}{X_3 + X_1}\right| < |\gamma| \left(\frac{V_{em}(ep \to e\pi^0 p) + V_{em}(en \to e\pi^0 n)}{4\sigma(\nu n \to \mu^- \pi^0 p)} \right)^{1/2} \simeq 0.45,$$

where the numerical estimate is again based on Δ dominance. Available data suggest that the nonresonant background in neutrino or photon reactions at the relevant energy interval is at most about 25 ~ 30%. Thus assuming

$$\left| \frac{X_3}{X_3+X_1} \right| \simeq 0.80,$$

we have

$$R_4 \gtrsim \left[\frac{3}{2} \left| \frac{X_3}{X_3+X_1} \right| - \frac{1}{2} - |y| \left(\frac{V_{em}(e\bar{N} \to e\pi^0 \bar{N})}{2\sigma(\omega n \to \rho^- \pi^0 p)} \right)^{1/2} \right]^2 \simeq 0.06. \tag{15}$$

This lower limit is physically unrealistic since it assumes a destructive interference of the I = 3/2 and 1/2 amplitudes. A more realistic bound is obtained if we assume X_1 and X_3 to be 90° out of phase, and allow $|X_1|^2/(|X_1|^2 + |X_3|^2)$ to be as large as 0.30. In this way we have

$$R_4 \geq \left[\left(1 - \frac{3}{4} \frac{|X_1|^2}{|X_3|^2+|X_1|^2}\right)^{1/2} - |y| \left(\frac{V_{em}(e\bar{N} \to e\pi^0 \bar{N})}{2\sigma(\omega n \to \rho^- \pi^0 p)} \right)^{1/2} \right]^2 \simeq 0.19. \tag{16}$$

This is to be compared with $R \leq 0.14$ (90% confidence level).

To compare the procedure employed here to that of part 1, we estimate a lower bound for the ratio

$$R_5 \equiv \frac{\sigma(\omega p \to \omega \pi^+ n) + \sigma(\omega n \to \omega \pi^- p)}{2\sigma(\omega p \to \rho^- \pi^+ p)}.$$

In place of (14), we find

$$R_5 \geq \frac{1}{9}\left[\left|\frac{X_5+X_1}{X_3}\right| - \left|\frac{\gamma \epsilon_V}{X_3}\right|\right]^2 + \frac{4}{9}|\gamma \epsilon_5|^2. \tag{17}$$

A conservative estimate of the amplitudes then yields

$$R_5 \geq \frac{1}{9}\left[1 - |\gamma|\left(\frac{9}{4}\,\frac{V_{em}(ep \to e\pi^+ n) + V_{em}(en \to e\pi^- p)}{\sigma(\nu p \to \mu^- \pi^+ p)}\right)^{1/2}\right]^2 \simeq 0.03, \tag{18}$$

which should be compared with the estimate (11) for R_3. By both techniques, we have been able to lower the bound from 0.11 to 0.03 by including both the I = 1/2 and I = 3/2 contributions as opposed to the latter along. The present experimental limit[11] for R_3 is 0.08 ± 0.04.

B. INCLUSIVE PROCESS

Reaction (2) for inclusive weak pion production is considerably more involved than (1) in that many more amplitudes appear since the unknown states include combinations of I = 1/2, 3/2, and 5/2. The procedure used in Section A2 is unreliable here; however, one can still relate the neutral current reactions to the charged current reactions as in A1.

The results obtained for the structure functions are similar to, but more complex than, those of Eqs. (5) and (6). We merely state the results obtained for the bounds analogous to (7) and (8) after averaging over proton and neutron targets:

$$R_1' \equiv \frac{\sigma(\nu p \to \nu\pi^+ X^0) + \sigma(\nu n \to \nu\pi^- X^+) + \sigma(\nu p \to \nu\pi^- X^{++}) + \sigma(\nu n \to \nu\pi^+ X^-)}{\sigma(\nu p \to \mu^-\pi^0 X^{++}) + \sigma(\nu n \to \mu^-\pi^0 X^+)}$$

$$\geq \left[1 - 2(\sin^2\theta_W) r_1^{1/2}\right]^2, \tag{19}$$

and

$$R_2' \equiv \frac{2\sigma(\nu\bar{N} \to \nu\pi^0\bar{X}) + \sigma(\nu\bar{N} \to \nu\pi^-\bar{X}) + \sigma(\nu\bar{N} \to \nu\pi^+\bar{X})}{2\sigma(\nu\bar{N} \to \mu^-\pi^+\bar{X}) + 2\sigma(\nu\bar{N} \to \mu^-\pi^-\bar{X})}$$

$$\geq \frac{1}{2}\left[1 - 2(\sin^2\theta_W) r_2^{1/2}\right]^2, \tag{20}$$

where

$$r_1 \equiv \frac{V_{em}(ep \to e\pi^+ X^0) + V_{em}(en \to e\pi^- X^+) + V_{em}(ep \to e\pi^- X^{++}) + V_{em}(en \to e\pi^+ X^-)}{2\sigma(\nu p \to \mu^-\pi^0 X^{++}) + 2\sigma(\nu n \to \mu^-\pi^0 X^+)}$$

and

$$r_2 \equiv \frac{2V_{em}(e\bar{N} \to e\pi^0\bar{X}) + V_{em}(e\bar{N} \to e\pi^+\bar{X}) + V_{em}(e\bar{N} \to e\pi^-\bar{X})}{2\sigma(\nu\bar{N} \to \mu^-\pi^+\bar{X}) + 2\sigma(\nu\bar{N} \to \mu^-\pi^-\bar{X})}.$$

The test of these bounds on the inclusive weak pion production cross sections awaits considerably more data from the electroproduction and weak charged current production processes.

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